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Elementary Set-Theory (Continue)

Set theoretic Difference :- Let A, B be sets. The Set theoretic difference $A \setminus B$ (or $A - B$) is defined by $x \in A\setminus B$ iff $x \in A$ and $x \notin B$.

Theorem :- $A \setminus (B \cup C) = (A \setminus B) \cap (A \setminus C)$

Proof :- Let x be arbitary

Then $x \in A \setminus (B \cup C)$ iff $x \in A$ and $x \notin (B \cup C)$

$\Leftrightarrow$ $x \in A$ and not ($x \in B$ or $x \in C$)

$\Leftrightarrow$ $x \in A$ and ($x \notin B$ and $x \notin C$)

$\Leftrightarrow$ ($x \in A$ and $x \notin B$) and ($x \in A$ and $x \notin C$)

$\Leftrightarrow$ $x \in (A \setminus B)$ and $x \in (A \setminus C)$

$\Leftrightarrow$ $x \in (A \setminus B) \cap (A \setminus C)$. proved.

Provisional definition of function :- Let A, B be sets. Then f is a function from A to B written $f : A \to B$, if f assigns a unique element $b \in B$ to each $a \in A$.

Basic idea to consider $f : R \to R$ defined by $f(x) = x^2$. To generalize we shall identify f with its graph. To generalize this to arbitary sets A and B we first need the concept of an ordered pair. I.E, a mathematical object $\langle a, b \rangle$ satisfying $(a,b) = (c,d)$ iff $a = c$ and $b = d$. In particular, $(1,2) \neq \{1,2\}$.

Cartesian Product :- If A, B are sets, then their Cartesian Product $A \times B = \{(a,b) \mid a \in A, b \in B\}$.

Function :- f is a function from A to B iff $f \subseteq A \times B$ and for each $a \in A$, there exists a unique $b \in B$ such that $(a,b) \in f$.

In this case, the unique value b is called the value of f at a, and we write $f(a) = b$.

It only remains to define (a,b) in terms of set theory.

Ordered pair :- $(a,b) = \{\{a\}, \{a,b\}\}$

Theorem :- $(a,b) = (c,d)$ iff $a = c$ and $b = d$.

Proof :- Clearly if $a = c$ and $b = d$, then
$$(a,b) = \{\{a\}, \{a,b\}\} = \{\{c\}, \{c,d\}\} = (c,d).$$

1. Suppose $a = b$ Then
$$\{\{a\}, \{a,b\}\} = \{\{a\}, \{a,a\}\} = \{\{a\}, \{a\}\} = \{\{a\}\} = \{\{c\}, \{c,d\}\}$$
we must have $\{a\} = \{c\}$ and $\{a\} = \{c,d\}$. so $a = c = d$ in particular, $a = c$ and $b = d$.

2. Suppose $c = d$ The similarly $a = b = c$ so $a = c$ and $b = d$

3. Suppose $a \neq b$ and $c \neq d$

Since, $\{\{a\}, \{a,b\}\} = \{\{c\}, \{c,d\}\}$

we must have $\{a\} = \{c\}$ or $\{a\} = \{c,d\}$

The latter is clearly impossible, so $a = c$.

Similarly, $\{a,b\} = \{c,d\}$ or $\{a,b\} = \{c\}$.

The latter is clearly impossible,

and as $a = c$, then $b = d$.

* It is almost never necessary in a mathematical proof to remember that a function is literally a set of ordered pairs.